Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 1: Pure Mathematics

PMT Mock 3

Time: 2 hours	Paper Reference(s) 9MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. The point $P(2, -3)$ lies on the curve with equation $y = \mathrm{f}(x)$.
State the coordinates of the image of P under the transformation represented by the curve

a. $y = |\mathrm{f}(x)|$

(1)

b. $y = \mathrm{f}(x - 2)$

(1)

c. $y = 3\mathrm{f}(2x) + 2$

(2)

(Total for Question 1 is 4 marks)

2. $$f(x) = (2x - 3)(x - k) - 12$$

where k is a constant.

a. Write down the value of $f(k)$

(1)

When $f(x)$ is divided by $(x + 2)$ the remainder is -5

b. find the value of k.

(2)

c. Factorise $f(x)$ completely.

(3)

(Total for Question 2 is 6 marks)

3. A circle C has equation

$$x^2 - 22x + y^2 + 10y + 46 = 0$$

a. Find

i. the coordinates of the centre A of the circle

ii. the radius of the circle.

(3)

Given that the points $Q(5,3)$ and S lie on C such that the distance QS is greatest,

b. find an equation of tangent to C at S, giving your answer in the form

$ax + by + c = 0$, where a, b and c are constants to be found.

(3)

(Total for Question 3 is 6 marks)

4. a. Express $\lim_{dx \to 0} \sum_{0.2}^{1.8} \frac{1}{2x} \delta x$ as an integral.

(1)

b. Hence show that

$$\lim_{dx \to 0} \sum_{0.2}^{1.8} \frac{1}{2x} \delta x = \ln k$$

where k is a constant to be found.

(2)

(Total for Question 4 is 3 marks)

5. A scientist is studying a population of lizards on an island and uses the linear model $P = a + bt$ to predict the future population of the lizard where P is the population and t is the time in years after the start of the study.

Given that

- The number of population was 900, exactly 5 years after the start of the study.
- The number of population was 1200, exactly 8 years after the start of the study.

a. find a complete equation for the model.

(4)

b. Sketch the graph of the population for the first 10 years.

(1)

c. Suggest one criticism of this model.

(1)

(Total for Question 5 is 6 marks)

6.

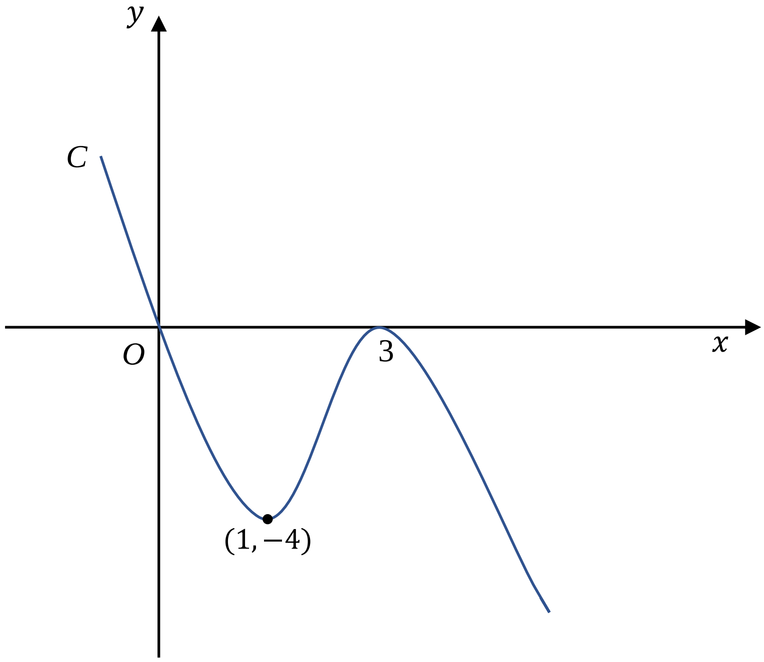

Figure 1

The figure 1 shows sketch of the curve C with equation $y = \mathrm{f}(x)$.

$$\mathrm{f}(x) = ax(x - b)^2, x \in R$$

where a and b are constants.

The curve passes through the origin and touches the x-axis at the point (3,0).

There is a minimum point at (1,–4) and a maximum point at (3,0).

a. Find the equation of C.

(3)

b. Deduce the values of x for which

$$\mathrm{f}'(x) > 0$$

(1)

Given that the line with equation $y = k$, where k is a constant, intersects C at exactly one point,

c. State the possible values for k.

(2)

(Total for Question 6 is 6 marks)

7. (i) Given that a and b are integers such that

$$a + b \text{ is odd}$$

Use algebra to prove by contradiction that at least one of a and b is odd.

(3)

(ii) A student is trying to prove that

$$(p+q)^2 < 13p^2 + q^2 \qquad \text{where } p < 0$$

The student writes:

$$p^2 + 2pq + q^2 < 13p^2 + q^2$$

$$2pq < 12p^2$$

$$\text{so as } p < 0 \qquad 2q < 12p$$

$$q < 6p$$

a. Identify the error made in the proof.

(1)

b. Write out the correct solution.

(1)

(Total for Question 7 is 5 marks)

8.

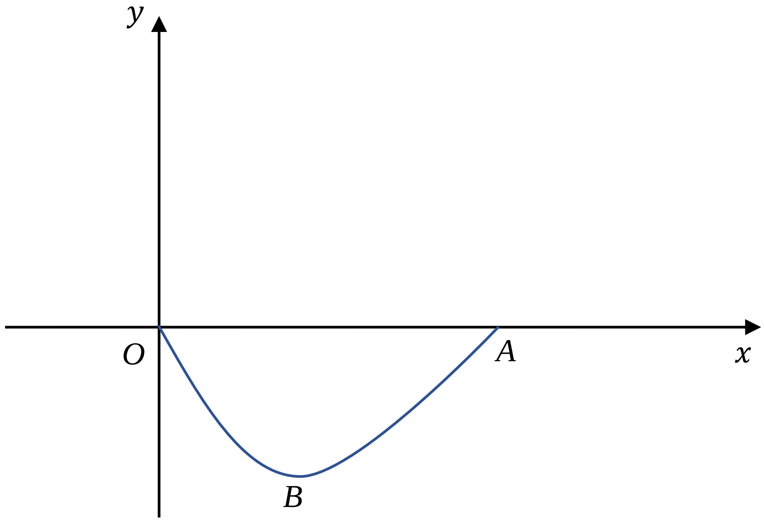

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where $x \in R$, $x > 0$

$$\mathrm{f}(x) = (0.5x - 8)\ln(x + 1) \qquad 0 \leq x \leq A$$

a. Find the value of A.

(1)

b. Find $\mathrm{f}'(x)$

(2)

The curve has a minimum turning point at B.

c. Show that the x-coordinate of B is a solution of the equation

$$x = \frac{17}{\ln(x + 1) + 1} - 1$$

(2)

d. Use the iteration formula

$$x_{n+1} = \frac{17}{\ln(x_n + 1) + 1} - 1$$

with $x_0 = 5$ to find the values of x_1 and the value of x_6 giving your answers to three decimal places.

(3)

(Total for Question 8 is 8 marks)

9.

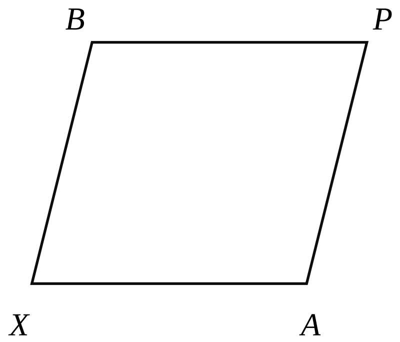

Figure 3

Figure 3 shows a sketch of a parallelogram *XAPB*.

Given that $\overrightarrow{OX} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$

a. Find the coordinates of the point *P*.

(3)

b. Show that *XAPB* is a rhombus.

(2)

c. Find the exact area of the rhombus *XAPB*.

(3)

(Total for Question 9 is 8 marks)

10. The figure 4 shows the curves $f(x) = A - Be^{-0.5x}$ and $g(x) = 26 + e^{0.5x}$

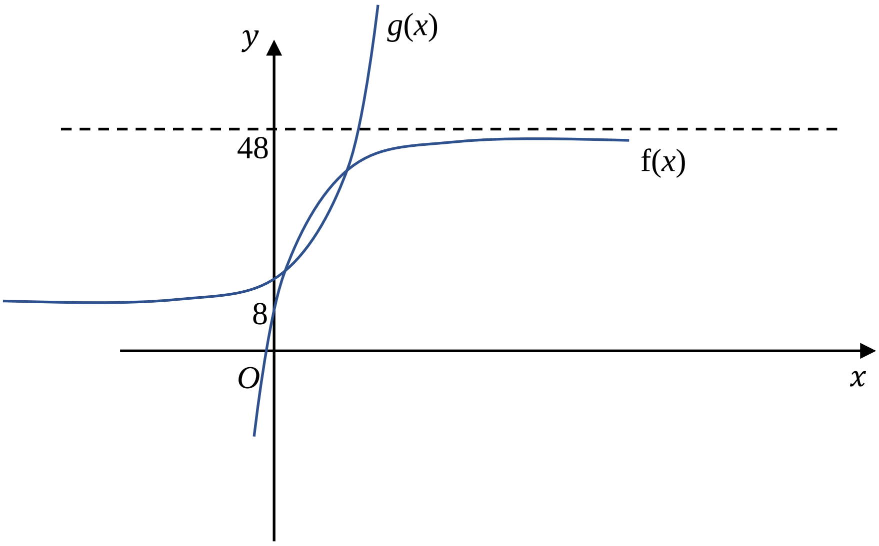

Figure 4

Given that $f(x)$ passes through (0,8) and has an horizontal asymptote $y = 48$

a. Find the values of A and B for $f(x)$

(3)

b. State the range of $g(x)$

(1)

The curves $f(x)$and $g(x)$ meet at the points C and D

c. Find the x-coordinates of the intersection points C and D, in the form $\ln k$, where k is an integer.

(4)

(Total for Question 10 is 8 marks)

11.

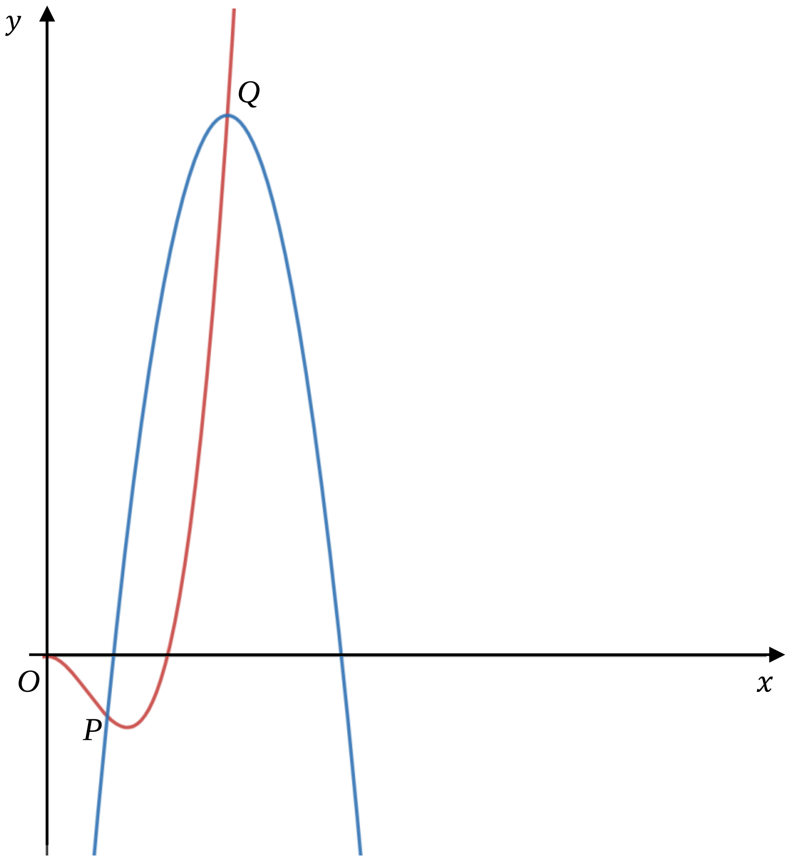

Figure 5

The figure 5 shows part of the curves C_1 and C_2 with equations

$$C_1: y = x^3 - 2x^2 \qquad x > 0$$

$$C_2: y = 9 - \frac{5}{2}(x - 3)^2 \qquad x > 0$$

The curves C_1 and C_2 intersect at the points P and Q.

a. Verify that the point Q has coordinates (3,9)

(1)

b. Use algebra to find the coordinates of the point P.

(6)

(Total for Question 11 is 7 marks)

12.

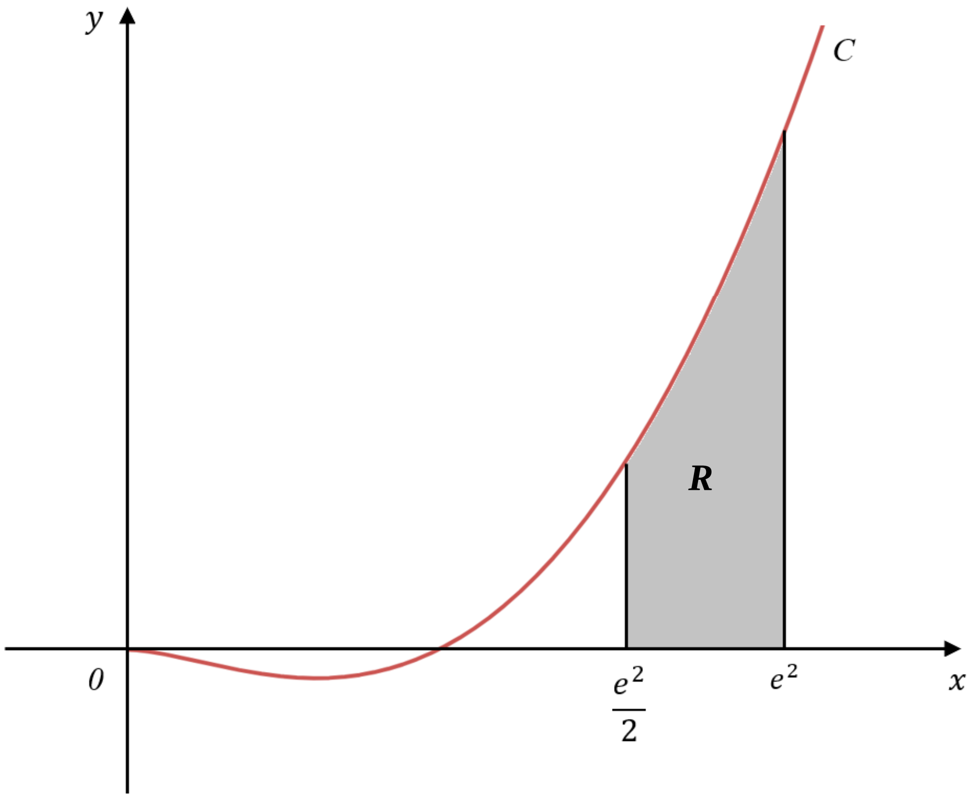

Figure 6

The figure 6 shows a sketch of the curve with equation

$$y = x^2 \ln 2x$$

The finite region R, shown shaded in figure 5, is bounded by the line with equation $x = \frac{e^2}{2}$, the curve C, the line with equation $x = e^2$ and the x-axis.

Show that the exact value of the area of region R is $\frac{e^6}{72}(35 + 24 \ln 2)$.

(Total for Question 12 is 5 marks)

13. A construction company had a 30-year programme to build new houses in Newtown. They began in the year 1991 (Year 1) and finished in 2020 (Year 30).
The company built 120 houses in year 1, 140 in year 2, 160 houses in year 3 and so on, so that the number of houses they built form an arithmetic sequence.
A total of 8400 new houses were built in year *n*.

a. Show that

$$n^2 + 11n - 840 = 0$$

(2)

b. Solve the equation

$$n^2 + 11n - 840 = 0$$

and hence find in which year 8400 new houses were built.

(2)

(Total for Question 13 is 4 marks)

14. Given that

$$2\cos(x+60)^0 = \sin(x-30)^0$$

a. Show, without using a calculator, that

$$\tan x = \frac{\sqrt{3}}{3}$$

(4)

b. Hence solve, for $0 \leq \theta < 360^0$

$$2\cos(2\theta+90)^0 = \sin(2\theta)^0$$

(4)

(Total for Question 14 is 8 marks)

15.

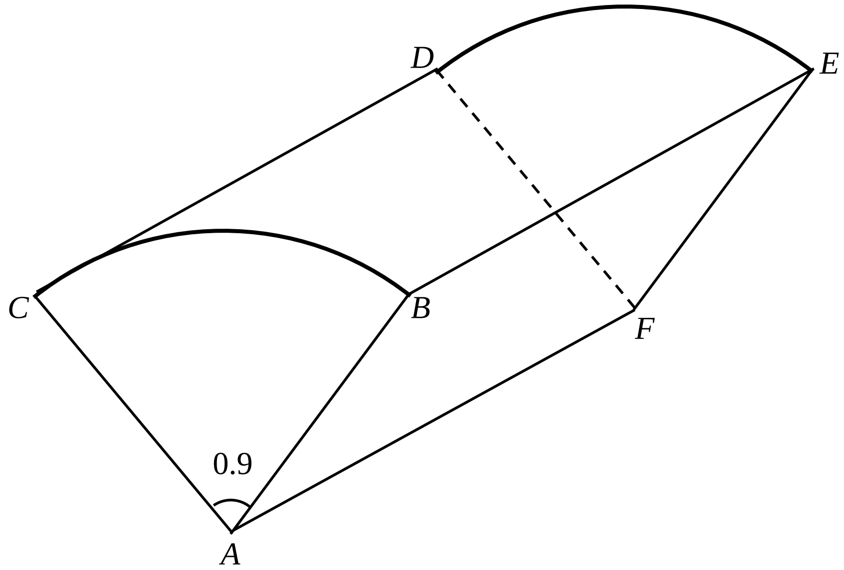

Figure 7

Figure 7 shows an open tank for storing water, *ABCDEF*. The sides *ACDF* and *ABEF* are rectangles. The faces *ABC* and *FED* are sectors of a circle with radius *AB* and *FE* respectively.

- $AB = FE = r$ cm
- $AF = BE = CD = l$ cm
- angle BAC = angle $EFD = 0.9$ radians

Given that the volume of the tank is 360 cm^3

a. show that the surface area of the tank, S cm^2, is given by

$$S = 0.9r^2 + \frac{1600}{r}$$

(4)

Given that r can vary

b. use calculus to find the value of r for which S is stationary.

(4)

c. Find, to 3 significant figures the minimum value of S.

(2)

(Total for Question 15 is 10 marks)

16.

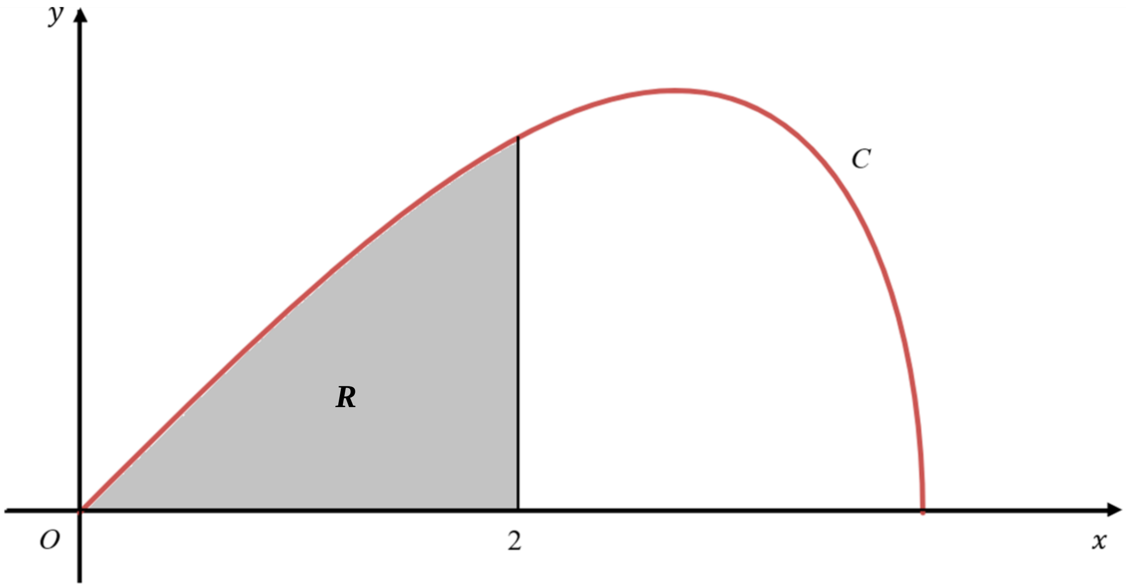

Figure 8

Figure 8 shows a sketch of the curve with parametric equations

$$x = 4\cos t \qquad y = 2\sin 2t \qquad 0 \leq t \leq \frac{\pi}{2}$$

where t is a parameter.

The finite region R is enclosed by the curve C, the x-axis and the line $x = 2$, as shown in Figure 7.

a. Show that the area of R is given by

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 16\sin^2 t\cos t \;\; \mathrm{d}t$$

(3)

b. Hence, using algebraic integration, find the exact area of R, giving in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(3)

(Total for Question 16 is 6 marks)